

# MAT 123 Final Exam

Part I (Type A)  
November 21, 2005

Student ID:

Name:

## NOTICE

1. On your OPSCAN form you should write your last name, first name and **Stony Brook ID** only. Bubble in the circles correspondingly. **DO NOT write anything in the BIRTHDATE and SPECIAL CODES box.** Instructions to fill in those boxes will be given to you in part II.
2. Do not fold your OPSCAN sheet as the machine will not be able to read it.
3. Do not mark anything on the bottom of the OPSCAN where black markers are printed.
4. The answers you mark on this exam paper will **NOT** count! You should make sure you have bubbled in all your answers on the OPSCAN.
5. You will only be allowed to work with your calculator for **the first 75 minutes**. After that, you will receive part II, which is to be worked without any use of calculators. After receiving part II you may still work on part I, but without the use of the calculator.

1. For which of the following value(s) does  $\sin \theta = 0.5$  hold?

I. $\theta = \frac{\pi}{6}$	II. $\theta = \frac{11\pi}{6}$
III. $\theta = \frac{17\pi}{6}$	

- ① I only                      ② I and II  
③ I and III                  ④ II and III  
⑤ I, II and III

2. Given that  $\sin \theta = \frac{4}{5}$  and  $\frac{\pi}{2} < \theta < \pi$ , find

$\cos \theta$  :

- ①  $-\frac{39}{20}$                                       ②  $\frac{39}{20}$   
③  $\frac{9}{5}$     ④  $-\frac{12}{5}$   
⑤ None of the above

3. Daniel put \$10,000 in a bank where the interest rate is 3% per year compounded monthly. If he withdraws the money after 20 years, how much money would he **gain**? (assume that he does not withdraw or deposit any money during that 20 years.)

- ① \$512.05            ② \$8,061.11  
 ③ \$8,140.18        ④ \$8,221.18  
 ⑤ \$8,207.54

4. Find the  $x$ -intercepts of the graph of the function  $f(x) = x^3 + 2x^2 - x - 2$  and choose the correct answer below.

- ① -2 only  
 ② -2, -1, 1 only  
 ③ 0 only  
 ④ 0, 1, -2 only  
 ⑤ There are no  $x$ -intercepts

5. Which of the following equation has a slanted asymptote and does not have any vertical asymptote?

- ①  $y = \frac{x^3}{x^2 + 1}$             ②  $y = \frac{x^3}{x^2 - 1}$   
 ③  $y = \frac{x^2}{x + 1}$             ④  $y = \frac{x}{x^2 + 1}$   
 ⑤ None of the above

6. In a bacteria culture, initially there were  $N=1000$  bacteria. Half an hour later, the population grew to  $N=1500$ . Assuming exponential growth, which of the following equations best describes the relation between number of bacteria  $N$  and time  $t$  (hours)?

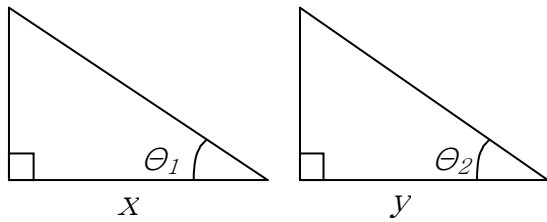
- ①  $N = 1000e^{\frac{\ln 2}{2}t}$   
 ②  $N = 1000e^{0.5(\ln 1500)t}$   
 ③  $N = 1000e^{2(\ln 1500)t}$   
 ④  $N = 1000e^{2(\ln 1.5)t}$   
 ⑤  $N = (1500 - 1000)e^{0.5t}$

7. The half life of the radioactive element plutonium-239 is 25,000 years. If 24g of plutonium-239 are initially present, how many grams are present after 50,000 years?

- ① 0g    ② 12g    ③ 6g    ④ 8g  
 ⑤ None of the above

8. What is the amplitude and the period of the graph of  $y = 2 \sin(3x - 5) + 6$  ?

- ① Amplitude = 8, Period =  $\frac{2\pi}{5}$   
 ② Amplitude = 2, Period =  $\frac{\pi}{3}$   
 ③ Amplitude = 8, Period =  $\frac{2\pi}{3}$   
 ④ Amplitude = 5, Period =  $\frac{\pi}{2}$   
 ⑤ None of the above



9. The base lengths of above triangles are  $x=5$  in. and  $y=4$  in. The angles are  $\theta_1=\pi/5$  rad and  $\theta_2=40^\circ$ . What is the sum of the lengths of the two triangles' hypotenuses? Round to the nearest tenth. ( Figures not drawn to scale. Notice that the two angles have different units!)

- ① 14.7
- ② 11.4
- ③ 13.9
- ④ 10.2
- ⑤ None of the above

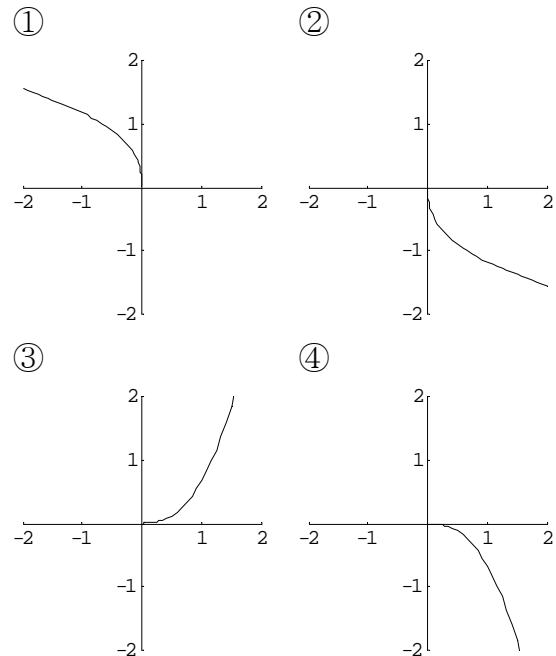
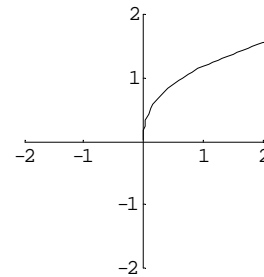
10. Find the average rate of change of the function  $f(x)=x+2\sin x$  from  $x_1=0$  to  $x_2=\frac{\pi}{4}$

- ①  $\frac{\pi}{4} + \frac{2}{\sqrt{2}}$
- ②  $1 + \frac{4\sqrt{2}}{\pi}$
- ③ 0
- ④  $1 + \frac{2}{\sqrt{2}}$
- ⑤ None of the above.

11. What is the (natural) domain of the function  $f(x)=\frac{1+\sqrt{x}}{\sqrt{5-x}}$ ?

- ①  $[0,5]$
- ②  $(0,5)$
- ③  $[0,5)$
- ④  $(0,5]$
- ⑤ None of the above

12. For the graph given below, indicate which of the graphs in figures ① ② ③ or ④ represents the correct inverse  $f^{-1}(x)$ .



- ⑤ None of the above

13. A rectangle of sides  $a$  and  $b$  has total perimeter 8. How much is the area when  $b$  is 3 times  $a$ ?

- ① 64    ② 16    ③  $64/3$     ④ 12  
 ⑤ None of the above

14. The annual subscription for Moose Town Daily costs \$10 and it has currently 250 subscribers. Recent analysis shows that for each dollar of subscription fee raised, the number of subscribers will drop by 9. What is the equation that correctly describes this analysis? ( $N$  is the number of subscribers and  $s$  is the annual subscription fee.)

- ①  $N = 260 - 10(s - 9)$   
 ②  $N = 250 - 9(s - 10)$   
 ③  $N = 1150 - 90s$   
 ④  $N = 259 - 10s$   
 ⑤ None of the above

15. What is the end behavior of the function  $f(x) = x(x-1)(2-x)(x-200)$ ?

- ① ↙ and ↘ (falls left and falls right)  
 ② ↖ and ↘ (rises left and falls right)  
 ③ ↙ and ↗ (falls left and rises right)  
 ④ ↖ and ↗ (rises left and rises right)  
 ⑤ The graph has a horizontal asymptote so it becomes flat at both ends.

16. Select all odd function(s) from below. (An odd function is a function having a point symmetry at the origin.)

I.  $(x^5 + x)(x^3 + x)$   
 II.  $\frac{x^3}{x^2 + 1}$                       III.  $\sin x$

- ① I only                      ② II and III  
 ③ I and III                ④ III only  
 ⑤ I, II and III

17. What is the horizontal asymptote of the function  $y = \frac{3x^2 - 2}{x^3 + 20}$ ?

- ①  $y = 0$                       ②  $x = -\sqrt[3]{20}$   
 ③  $x = -20$                 ④  $y = 3$   
 ⑤ None of the above

18. What is the  $y$ -intercept of the graph of

$$y = \frac{x \cos x}{x^2 + 1}?$$

- ①  $\frac{\pi}{2}$   
 ②  $-1$   
 ③ 1  
 ④ 0  
 ⑤ None of the above.

19.  $\ln a = 2$  and  $\ln b = 4$ . What is  $\log_a b$ ?

- ①  $\frac{1}{2}$     ② 2    ③ 1    ④ 4  
 ⑤ None of the above.

20. Solve the equation  $2\log_5 x = 1$  and choose the correct answer below.

- ①  $x = \sqrt{5}$  or  $x = \frac{1}{\sqrt{5}}$  only
- ②  $x = \frac{1}{\sqrt{5}}$  only
- ③  $x = \sqrt{5}$  only
- ④  $x = \frac{1}{2}$  only
- ⑤ None of the above

21. Write  $-3\ln(x+1) + 2\ln(x^2 + 2)$  into a single logarithm.

- ①  $\ln\left(\frac{(x-1)^3}{x^2+2}\right)$
- ②  $\ln\left(\frac{(x^2+2)^2}{(x-1)^{3/2}}\right)$
- ③  $\ln\left(\frac{2(x^2+2)^3}{3(x-1)}\right)$
- ④  $\ln((x^2+2)^2(x-1)^3)$
- ⑤ None of the above

22. Convert  $225^\circ$  to radians:

- ①  $\frac{8\pi}{3}$
- ②  $-\frac{4\pi}{3}$
- ③  $8\pi$
- ④  $\frac{2\pi}{3}$
- ⑤ None of the above

23. Find the slope of the tangent line to  $y = 2x^3 + 5x^2 - 3x + 1$  at  $x = -1$ .

- ① 5
- ② 7
- ③ -7
- ④ -2
- ⑤ None of the above.

24. What is the derivative of  $f(x) = \sqrt{x}$ ?

- ①  $\frac{1}{2}\sqrt{x}$
- ②  $\frac{1}{\sqrt{x}}$
- ③  $\frac{1}{2\sqrt{x}}$
- ④  $\sqrt{x}$
- ⑤ None of the above.

25. The inflection points of the function  $f(x) = 3x^5 - 5x^3 + 21$  are:

- ①  $x = 1, x = \frac{1}{\sqrt{2}}, x = -\frac{1}{\sqrt{2}}$  only
- ②  $x = -1, x = 0, x = 1$  only
- ③  $x = 0, x = \frac{1}{\sqrt{2}}, x = -\frac{1}{\sqrt{2}}$  only
- ④  $x = 0, x = 1$  only
- ⑤ None of the above.

# MAT 123 Final Exam

Part II (Type A)  
November 22, 2005

Student ID:

Name:

IMPORTANT : You should fill in and bubble in the first column of the BIRTH DATE - DAY box as follows. **Please leave the second box of DAY and the other boxes - MO. YR. and SPECIAL CODES blank.**

Part I	Part II	DAY	
Type A	Type A	0	
Type A	Type B	1	
Type B	Type A	2	
Type B	Type B	3	

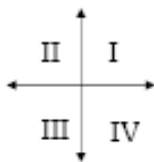
IF YOU MARK THIS WRONG, ALL YOUR ANSWERS WILL BE GRADED INCORRECTLY. PLEASE TAKE CARE TO WRITE THE CORRECT SPECIAL CODE.

26. What is the value of  $\sin\left(\frac{\pi}{3}\right)$ ?

- ①  $\frac{\sqrt{3}}{2}$                       ②  $\frac{1}{2}$   
③  $-\frac{1}{2}$                       ④  $\frac{\sqrt{2}}{2}$   
⑤ None of the above

27. If  $\sin\theta < 0$  and  $\sec\theta > 0$ , find the quadrant in which the angle  $\theta$  lies.

- ① Quadrant I  
② Quadrant III  
③ Quadrant II  
④ Quadrant IV



⑤ The terminal side of the angle lies on the coordinate axis.

28. Which of the following has the same value

as  $\sec\left(\frac{\pi}{6}\right)$ ?

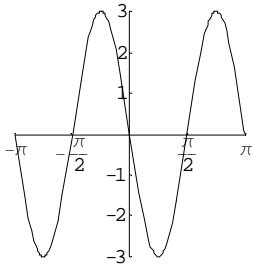
- ①  $\cos\frac{\pi}{6}$                       ②  $\csc\frac{\pi}{3}$   
③  $\sin\frac{\pi}{6}$                       ④  $\sec\frac{\pi}{3}$   
⑤ None of the above

29. Compute  $\sin\left(\tan^{-1}\sqrt{3}\right)$ :

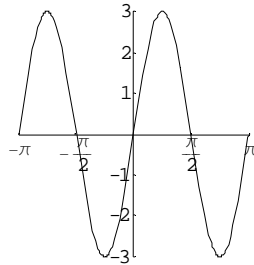
- ①  $\frac{\sqrt{2}}{2}$                       ②  $\frac{1}{\sqrt{3}}$   
③  $\frac{\sqrt{3}}{2}$                       ④  $\frac{1}{2}$   
⑤ None of the above

30. Which of the following is the graph of  $y = 3\sin(2x - \pi)$ ?

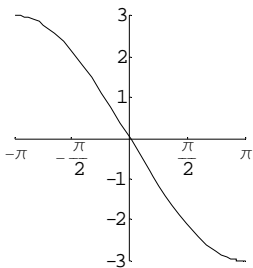
①



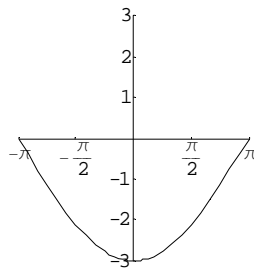
②



③



④



⑤ None of the above.

31. What is the equation of the slanted asymptote of  $f(x) = \frac{x^2 + 3x}{x+1}$ ?

①  $y = x + 2$

②  $y = 2x$

③  $y = 2x + 1$

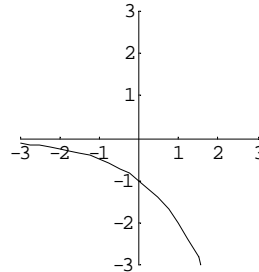
④ The function does not have a slanted asymptote.

⑤ None of the above

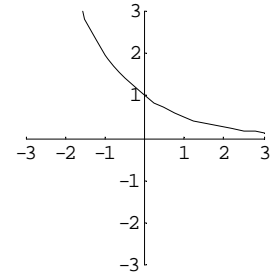
32. Which of the following graphs represents

the function  $f(x) = -\left(\frac{1}{2}\right)^x$ ?

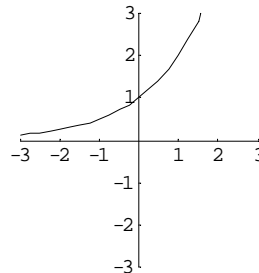
①



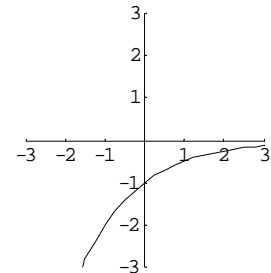
②



③



④



⑤ None of the above

33. Evaluate the expression  $\log_8\left(\frac{1}{\sqrt[3]{64}}\right)$  and

choose the correct answer.

① 6    ② -6    ③  $-\frac{2}{3}$     ④  $\frac{2}{3}$

⑤ None of the above

34. Select correct equation(s) from below.

I.  $\ln x + \ln y = \ln xy$

II.  $(\log x)^3 = 3 \log x$

III.  $\ln(x + y) = \ln x + \ln y$

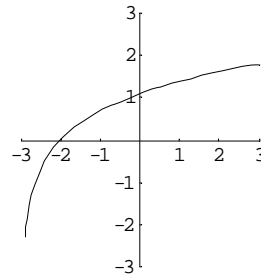
- ① I only            ② I and III  
 ③ I and II        ④ II and III  
 ⑤ I, II and III

35. Solve the following equation for  $x$ :

$$e^{2x} - 8e^x + 7 = 0$$

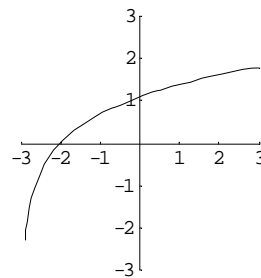
- ①  $x = 1$   
 ②  $x = e, \ln 7$   
 ③  $x = \ln \frac{7}{e}$   
 ④  $x = -1, -\ln 7$   
 ⑤ None of the above

36. The graph of  $y = f(x)$  is shown below.

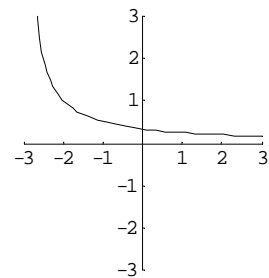


Which of the following is the graph of  $y = f'(x)$ ?

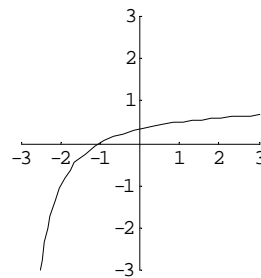
①



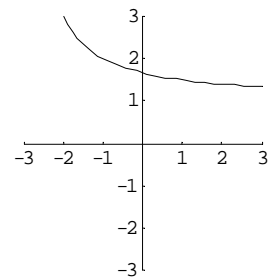
②



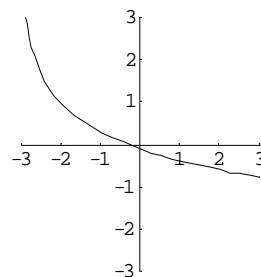
③



④



⑤





37. Let  $f(x) = x^2 - 5x + 6$ . Find all critical point(s) of  $f(x)$ .

- ①  $x = 3, x = \frac{5}{2}$  only
- ②  $x = 3, x = 2$  only
- ③  $x = 2$  only
- ④  $x = \frac{5}{2}$  only
- ⑤ None of the above.

38.  $f(x) = x^3 + 2x - 1$ . Select all correct statement(s) from below.

- I. The point  $(0, -1)$  on the graph of  $f(x)$  is the only point where the concavity changes from concave down to concave up.
- II.  $f(x)$  has no local maximum or local minimum.
- III.  $f(x)$  is an increasing function on any interval.

(Note: local maximum/minimum is also called relative maximum/minimum in some books.)

- ① II and III                      ② I and II
- ③ I and III                        ④ I only
- ⑤ I, II and III

39. What is the maximum value of  $f(x) = x^3 - 12x$  in the interval  $[-3, 3]$ ?

- ① 6                                      ② 18
- ③ 16                                    ④ 2
- ⑤ None of the above.

40. In which interval(s) is the function

$f(x) = \frac{2}{3}x^3 + \frac{x^2}{2} - 3x + 1$  concave up/down?

- ① Concave up:  $(-\frac{1}{4}, \infty)$ ,  
Concave down:  $(-\infty, -\frac{1}{4})$
- ② Concave up:  $(\frac{1}{4}, \infty)$ ,  
Concave down:  $(-\infty, \frac{1}{4})$
- ③ Concave up:  $(1, \infty)$  and  $(-\infty, -\frac{3}{2})$   
Concave down:  $(-\frac{3}{2}, 1)$
- ④ Concave up:  $(1, \infty)$ ,  
Concave down:  $(-\frac{3}{2}, 1)$
- ⑤ None of the above.